Exercise 225

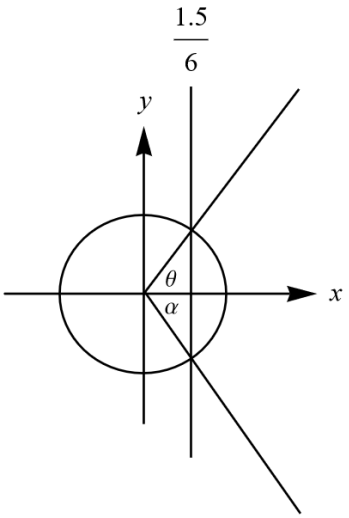
An object moving in simple harmonic motion is modeled by the function $s(t) = -6\cos\left(\frac{\pi t}{2}\right)$, where s is measured in inches and t is measured in seconds. Determine the first time when the distance moved is 4.5 in.

Solution

Notice that the object is at $s = -6$ when $t = 0$. When it moves 4.5 inches, then, it's at $s = -1.5$.

$$-1.5 = -6\cos\left(\frac{\pi t}{2}\right)$$

$$\frac{1.5}{6} = \cos\left(\frac{\pi t}{2}\right)$$



There are two angles, α and θ, that give 1.5/6 after taking the cosine. Taking the arccosine of 1.5/6 on the calculator gives θ. α is $2\pi - \theta$.

$$\alpha = 2\pi - \cos^{-1}\left(\frac{1.5}{6}\right) = \frac{\pi t}{2} \qquad \text{or} \qquad \theta = \cos^{-1}\left(\frac{1.5}{6}\right) = \frac{\pi t}{2}$$

$$\frac{2}{\pi}\left[2\pi - \cos^{-1}\left(\frac{1.5}{6}\right)\right] = t \qquad \text{or} \qquad \frac{2}{\pi}\cos^{-1}\left(\frac{1.5}{6}\right) = t$$

$$3.16 \approx t \qquad \text{or} \qquad 0.839 \approx t$$

The object moves 4.5 inches after about 0.839 seconds.